

Problem 9

In the following problems, find the limit of the given sequence as $n \rightarrow \infty$.

$$n \sin(1/n)$$

Solution

Take the limit as $n \rightarrow \infty$, using l'Hôpital's rule where it's appropriate.

$$\begin{aligned} \lim_{n \rightarrow \infty} n \sin(1/n) &= \lim_{n \rightarrow \infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \\ &\stackrel{\frac{0}{0}}{\text{H}} \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} \\ &= \lim_{x \rightarrow 0} \frac{\cos x}{1} \\ &= \cos 0 \\ &= 1 \end{aligned}$$